$=\frac{1}{4}\sqrt{214657\frac{1}{3}}=115.8278+$ ft.

1. B. $OE=\sqrt{(BE^2+OB^2)}=\sqrt{[(\frac{1}{4}\sqrt{214657\frac{1}{3}})^2+40^2]}$,
$=\sqrt{(13416\frac{1}{2}+1600)}=\sqrt{15016\frac{1}{12}}=122.5402+$ ft. $=$ the length of the ladder.

III. $\therefore$

1. $111.8796+$ ft. $=$ the distance from base of the ladder to the base of the tower $FC$,
2. $118.8111+$ ft. $=$ the distance from the base of the ladder to the base of the tower $AD$.
3. $115.8278+$ ft. $=$ the distance from the base of the ladder to the base of the tower $BE$, and
4. $122.5402+$ ft. $=$ the length of the ladder.

[From *Finkel's Mathematical Solution Book*, *p.* 299.]

[NOTE.—This method of solution may be easily extended to the more general case, viz., when the triangle is scalene.]

# ALGEBRA.

Conducted by J. M. COLAW, Monterey, Va. All contributions to this department should be sent to him.

## SOLUTIONS OF PROBLEMS.

**44. Proposed by LEONARD E. DICKSON, M. A., Fellow in Mathematics, University of Chicago.**

Find the general term in the series 1, 3, 10, 35, 126, 462, 1716, 6435, 24310, ...., which plays a remarkable part in some recent theorems in my Theory of Regular Polygons.

**Solution by the PROPOSER.**

This series is a "diagonal" in the Triangle of Pascal, as shown in the following table:—

| $C$ | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | |
|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 1 | | | | | | | | | |
| 1 | 1 | **1** | | | | | | | | |
| 2 | 1 | 2 | 1 | | | | | | | |
| 3 | 1 | 3 | **3** | 1 | | | | | | |
| 4 | 1 | 4 | 6 | 4 | 1 | | | | | |
| 5 | 1 | 5 | 10 | **10** | 5 | 1 | | | | |
| 6 | 1 | 6 | 15 | 20 | 15 | 6 | 1 | | | |
| 7 | 1 | 7 | 21 | 35 | **35** | 21 | 7 | 1 | | |
| 8 | 1 | 8 | 28 | 56 | 70 | 56 | 28 | 8 | 1 | |
| 9 | 1 | 9 | 36 | 84 | 126 | **126** | 84 | 36 | 9 | 1 |

Since the $mth$ term in the series lies at the intersection of column $m$ with

row $(2m-1)$, it is $\dfrac{(2m-1)(2m-2)\ldots.(2m-m)}{1.2.3\ldots.m}$

$$=\frac{(2m-1)(2m-2)\ldots.(m+1)}{1.2.3\ldots.(m-1)}=\frac{(2m)!}{2(m\,!)^2}.$$

The part played by the series in the Theory of Regular Polygons is indicated in an article in the current issue of the *Annals of Mathematics.*

Also solved by *A. H. BELL*, and *Professor J. F. W. SCHEFFER.*

45. **Proposed by WILLIAM HOOVER, A. M., Ph. D., Professor of Mathematics and Astronomy, Ohio University, Athens, Ohio.**

Find $x$ from $\cos^{-1}\dfrac{1-x^2}{1+x^2}+\tan^{-1}\dfrac{2x}{1-x^2}=\dfrac{4\pi}{3}.$

**I. Solution by T. W. PALMER, A. M., Professor of Mathematics, University of Alabama; H. W. DRAUGHON, Olio, Mississippi; A. L. FOOTE, C. E., Middlebury, Connecticut; and the PROPOSER.**

Let $\tan\theta=x$. Then $\tan 2\theta=\dfrac{2\tan\theta}{1-\tan^2\theta}$, $=\dfrac{2x}{1-x^2}$; and $\dfrac{1-x^2}{1+x^2}$

$$=\frac{1-\tan^2\theta}{1+\tan^2\theta},\ =\cos^2\theta(1-\tan^2\theta),=(\cos^2\theta-\sin^2\theta),=\cos 2\theta.$$

$$\therefore\ \cos^{-1}\frac{1-x^2}{1+x^2}+\tan^{-1}\frac{2x}{1-x^2},\ =\cos^{-1}\cos 2\theta+\tan^{-1}\tan 2\theta=\frac{4\pi}{3}.$$

$$\therefore\ 2\theta+2\theta,=4\theta,\ =\frac{4\pi}{3},\quad \therefore\ \theta=\frac{\pi}{3},=60^\circ.$$

$$\therefore\ x=\tan\theta,=\tan\frac{\pi}{3},=\tan 60^\circ,=\sqrt{3}.$$

$\therefore\ x=\sqrt{3}.$

**II. Solution by JOHN B. FAUGHT, A. B., Indiana University, Bloomington, Indiana; J. A. JOHNSON, Jr., Student of the Sophomore Class, University of Mississippi; P. S. BERG, Apple Creek, Ohio; and J. W. WATSON, Middle Creek, Ohio.**

$$\text{Since }\tan^{-1}\frac{2x}{1-x^2}=\cos^{-1}\frac{1}{\sqrt{1+\dfrac{4x^2}{(1-x^2)^2}}}=\cos^{-1}\frac{1-x^2}{1+x^2};$$

$$\therefore\ 2\cos^{-1}\frac{1-x^2}{1+x^2}=\frac{4}{3}\pi,\text{ or }\cos^{-1}\frac{1-x^2}{1+x^2}=\frac{2}{3}\pi=\cos^{-1}(-\tfrac{1}{2}).$$

$$\therefore\ \frac{1-x^2}{1+x^2}=-\tfrac{1}{2},\text{ whence }x^2=3,\text{ and }x=\pm\sqrt{3}.$$

Alfo solved by *F. P. MATZ, J, SCHEFFER, C. D. SCHMITT, E. L. SHERWOOD, M. C. STEVENS, G. B. M. ZERR;* and ————